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# Highlights

## July 19, 1987, Ithaca, New York

## Life in Mathematics
## (Interview 2, Parts 1 and 2, also includes highlights from May 22nd, 1987)

E. D. How did you get interested in mathematics?

M. F. As a child I lived in close proximity of the Moscow University and the Lenin Library. The first book on mathematics that drew my attention was the *Selected Problems and Theorems of Elementary Mathematics* by Shklyarskii and others. I looked through a few problems in this book and decided to join the math circle at the University run by Sasha Krylov and Kolya Bakhvalov.[1] I attended it from the seventh to the tenth grade, and I liked it a lot.

E. D. Math circles provide a wonderful experience for students and instructors alike.

M. F. The early 50s was the time when math circles flourished. They were founded on pure enthusiasm, without any outside pressure (in contrast to the situation in Moscow today). Among the members of our circle were Kirillov, Arnold, Tutubalin and Vinberg. My school years were closely entwined with the activity of that circle. From time to time I also attended another circle run by Arkady Onischick and Nikita Vvedenskaya.

E. D. What is Sasha Krylov is doing now?

M. F. He defended his Doctor of Science dissertation and works in the Institute of the Earth Physics.

E. D. What about your achievements in the mathematical Olympiads?

M. F. I enjoyed taking part in them and often won prizes, although never the highest one.

E. D. What was the highest prize you got?

M. F. I won the second prize a few times. I never won the first prize. The first prize, as a rule, went to Kirillov. I think Arnold also never received the first prize.

E. D. Did you have any problems getting into the university?

[1] http://en.wikipedia.org/wiki/Nikolai_Sergeevich_Bakhvalov

M. F. No. The year I took my entrance exams was a good year. I took seven exams.

E. D. This is because you didn't graduate from high school with a medal, right?

M. F. Yes, I didn't have one, but I scored 35 points out of 35 in the exams.

E. D. In other words, you scored 5 points in each exam. [2]

M. F. Correct. There was, however, a funny situation when I came to campus to verify that my name was on the list of admitted students. I was pretty confident that I got in because few people score 35. Surprisingly, my name was not on the list. The omission proved to be a simple mistake. So I was admitted to Mekhmath without any problems. That year the Chair of the admissions committee was Pavel Sergeyevich Alexandrov.[3]

E. D. It's very unlikely that he discriminated against the examinees.

M. F. Quite the opposite. When I was taking my oral exam in Russian language and literature, he entered the examination room and asked the examiners not to be too harsh to those who got 5 in mathematics since there were very few people who scored 5 points in mathematics. I scored 5 points in Russian language and literature as well. This is how I got into Mekhmath.

E. D. So one could say the beginning of your career was unclouded.

M. F. I guess so. One could say that most of my career was relatively unclouded.

E. D. What did you do in your first and second year of university?

M. F. Somewhere toward the end of my first year I joined your seminar. Most top students from our class, including Kirillov, Arnold, Vinberg, were in it as well. Sometimes the atmosphere was quite challenging. You would call a random student to the blackboard and ask him to solve a problem. I remember that one time you called on a student, and, when he couldn't provide an answer to your question, you referred to a Pavlov's experiment with an ape that was challenged to fetch a banana by connecting two sticks.

E. D. Yes, I wrote two statements on the blackboard and asked a student to combine them in order to obtain a stated result.

M. F. In your seminar I started to work on a term paper.

E. D. What was it about?

M. F. It was about the relation between Lie groups and Lie algebras.

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[2] 5 points = A.

[3] http://en.wikipedia.org/wiki/Pavel_Alexandrov

E. D. In my opinion every research mathematician should be familiar with this relation.

M. F. I agree. So I proved that the logarithm maps a neighborhood of the identity in a group to a Lie algebra. This was my term paper in the second year. Arkady Onischik served as my reviewer. In the third year I was writing a term paper on invariant measures on symmetric spaces. I found these problems difficult. I put a lot of effort into this calculation only to discover that this was already done by Cartan.[4] I was very upset.[5] If only I had known, I would have simply consulted his work. I had put a lot of effort into this problem. This was partly the reason why in my fourth year my interests veered toward the theory of probability.

E. D. Was it around 1954 or 55?

M. F. Yes, 1955 or 56.

E. D. Well, in 1955-56 I already began a series of works on Markov's processes.

M. F. In my fourth year, you left for China and you asked Girsanov[6] to be my adviser until you are back. I wrote a term paper on stochastic equations.

E. D. It must have been an important landmark in your career.

M. F. It was my first published paper. Later I incorporated parts of it into my diploma project.

E. D. Girsanov was a brilliant mathematician.

M. F. He also was a very nice person. We were more like friends than a teacher and a student.

E. D. Did you also work on diffusion processes with reflection?

M. F. Yes, I worked on processes with reflection at the beginning of my graduate studies. Later I wrote a dissertation on equations with small parameters. Then I stopped working on them for a while.

E. D. But later you returned to them.

M. F. Yes, and I have been working on them ever since.

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[4] en.wikipedia.org/wiki/Élie_Cartan

[5] Note of E. D. I believe that it is beneficial for a student to get by himself an important formula or a theorem even if it can be found in the literature. This pedagogical method was used regularly by A. Kronrod in teaching motivated students.

[6] http://en.wikipedia.org/wiki/Igor_Vladimirovich_Girsanov

E. D. What do you remember about your student years? It doesn't have to be related to my seminar.

M. F. There were a lot of talented students in our class. Many of them were former participants of math circles, people like Arnold and Kirillov.

E. D. For a while Arnold and Kirillov used to have fairly similar research interests.

M. F. Yes, what happened is this. Aside from your seminar there was also Vitushkin's seminar on the theory of function of a real variable, and in that seminar Arnold developed a keen interest in Hilbert's thirteenth problem. Vitushkin argued that not every continuous function of three variables can be represented as a superposition of continuous functions of two variables. Arnold claimed the opposite. I remember how at Vitushkin's birthday party each one vowed to prove his argument.

E. D. And who prevailed?

M. F. In a sense both of them did.[7]

E. D. Which one of your early papers was the most significant one in terms of personal validation as a scholar?

M. F. Like many young people, I went through a period when I had doubts as to whether I can become a good mathematician. In 1963 I wrote a paper on a priori bounds for degenerated elliptic equations, and since after that became more calm and confident. I was at the end of my graduate study.

E. D. Did you participate in the seminar on probability theory together with Skorokhod?

M. F. No, it took place before my time. Girsanov, Seregin, Yushkevich, and Khasminskii were some of the older people who participated in it.

E. D. For some time I taught a joint seminar with Dobrushin.

M. F. This was also before my time. Dobrushin lectured on random processes. The subject was taught by Kolmogorov in the first semester, but his lectures were too hard for us, and Dobrushin had to start all over again in the second semester. He was a very good lecturer. It was a difficult subject to teach. There were no textbooks on random processes at the time. Doob's book had just been published but it was not an easy read.

E. D. Then you defended your dissertation and were very lucky to stay in the department.

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[7] Arnold was right. See http://en.wikipedia.org/wiki/Hilbert's_thirteenth_problem

M. F. Yes, my career is exceptional in this way.

E. D. Not unique but quite unusual—let's put it this way. Since 1945 there were very few cases when a Jewish student has been hired by Mekhmat after defending his Ph. D. thesis.

M. F. A number of students in our class became good mathematicians: Kirillov, Arnold, Tutubalin, Shur, Vinberg. We were good friends, we celebrated holidays together. Most of us stayed in MSU as PhD students, and about eight were hired as Mekhmath faculty.

E. D. What about your teachers? Who had the most influence on you? Who were your first and second year teachers.

M. F. In my first year I took your algebra course and I must say you were a good lecturer. Analysis was taught by Tumarkin. He was a meticulous teacher but a bit boring.

E. D. I took classes with Tumarkin and Gelfand, and I was more impressed with Gelfand. I also took a class with Delone, a rather interesting character.

M. F. Delone taught a course in geometry, but I wasn't in his section. I was in Alexandrov's section.

E. D. Did you like Alexandrov as a teacher?

M. F. Not so much as a teacher but as a person. He taught analytical geometry which even then seemed a very strange field of mathematics. In class he wasn't concerned only with mathematics but expressed his views on all kinds of subjects.

We took a lot of classes with you. You taught required courses on algebra and probability theory. Olga Arsenyevna Oleinik taught partial differential equations and Pontryagin taught ordinary differential equations.

E. D. What were they like as teachers? Did they show interest in their students?

M. F. I don't recollect anything from Oleinik's lectures. Later I became interested in PDE. But my interest was not related to her lectures. Yet I do remember Arlen Il'in, who taught exercises for her course. He is a very good mathematician, although he probably did not live up to his full potential.

E. D. Olga Arsenyevna on the other hand made a good career, even though she did not achieve what she wanted: she is neither a member of the Academy nor a correspondent member.

M. F. She published a few solid works but ...

E. D. And then of course the tragedy of her life is her perennial rivalry with Olga Alexandrovna Ladyzhenskaya.

M. F. I always liked Olga Alexandrovna. This has more to do with her personality than with her research. I always enjoyed talking to her about mathematics. When I was close to defending my doctoral dissertation, she invited me to Leningrad to give a talk at her seminar. Actually, it was Smirnov's seminar, but at the time she already taken it over.

E. D. Ladyzhenskaya was a protégé of Smirnov, whereas Oleinik was a protégé of Petrovsky.

M. F. Smirnov was in attendance and considered it his duty to entertain the speakers during the break (a tradition which does not exist in Moscow; they know how to treat their guests). So Smirnov didn't find anything better than to tell me anecdotes about Kolmogorov.

E. D. Disrespectful ones, I presume.

M. F. Yes. Ladyzhenskaya suggested that I defend my dissertation in Leningrad. I was inclined to do that because the situation in Moscow was becoming intolerable. Eskin didn't pass his dissertation defense. Kolmogorov happened to be in Leningrad at the same time, and I told him that Ladyzhenskaya thought that I should defend in Leningrad and that she offered to be my opponent. Kolmogorov responded: 'Why do you need to associate with this extravagant woman?' So I had to follow Kolmogorov's advice and defended at Moscow University. My opponents were Skorokhod, Khasminskii and Oleinik.

E. D. Kolmogorov was always very protective of you.

M. F. I always felt his support. Even when I was transferring to the Chair of Biophysics, Kolmogorov invited me to his office and said that he was unable to help me to get a position of a professor at Mekhmath. When I decided to move to Biophysics, his letter to the rector was a key factor. Yes, Kolmogorov played an important role in all critical moments of my career.

I worked on various mathematical models in collaboration with biophysicists, but it never occurred to me that I could work at the Chair of Biophysics. However the situation in Mekhmath became repugnant. In 1977 the head of the Biophysics Chair Andrei Rubin[8] offered me a position there and everything worked out well. My transfer was the last

[8] http://www.biophys.msu.ru/personal/rubin/cv.htm

document signed by the rector Khokhlov. On the following day Khokhlov went to the mountains and died there.

E. D. People say he was an honest man.

M. F. Not only was he honest, but he had done a lot of good deeds.

E. D. You cannot say that about his successor, Logunov.

M. F. Logunov is not a good match for the university. In fact, he is not interested in it at all, at least not so far as research and instruction is concerned. He issues a lot of pointless administrative regulations. In general opinion he is not doing a good job .

E. D. What do you remember about Iszrail Moiseevich Gelfand?

M. F. Our paths crossed only once, when I wanted to submit a paper to the Functional Analysis.

E. D. Victor Kac told me that Gelfand refused to write a recommendation letter for him. He said that it would be unfair to give Victor an advantage over young American scholars who were as good as him and whose only shortcoming was that Gelfand didn't know them.

M. F. Sounds quite absurd. In my case everything went well. My paper was published in the first volume of the journal. I think it was 1966. I brought my paper to Agranovich who was the scientific secretary of the editorial board. He rejected it saying that the journal maintains a very high standards of papers, even though he didn't know me or have any clue as to what my article was about. So I went to Gelfand. Gelfand asked what my article was about. I said it was about quasilinear parabolic equations. He sent it for review, and it was eventually published.

I had a much worse experience with the *Matematicheskie Zametki*,[9] whose chief editor was Stechkin.[10] He rejected an article on the grounds that in it I noted that a certain statement could also be proved under different conditions. I was told that the journal publishes only complete proofs of mathematical statements. So I went to their office and said that I can simply cross out this remark. Stechkin, who happened to be there, said, "How come all of you are so good in finding loopholes!" He handed back my paper. I ended up publishing it in the *Probability Theory* I believe. Also, after I applied for an exit visa, all of my papers that were already in the process of publication were returned under various

[9] *Mathematical Notes.*
[10] http://en.wikipedia.org/wiki/Sergey_Stechkin

pretexts. One article for example was rejected supposedly because it contained a statement that was not proven. I also submitted a couple of notes to the *Doklady*. When they weren't published in due time, I called Prokhorov,[11] who was a member of the editorial board. He promised me to fix the problem, and indeed they appeared in the next issue.

E. D. Tell me about your relationship with other mathematicians.

M. F. I had a somewhat awkward encounter with Yuri Vladimirovich Linnik. Shortly before his death he was working on a certain statistical problem. He came to Moscow and invited me to his hotel, saying that he had a problem that he thought I could solve. At the time I was very keen on the subject of large deviations. So when he presented his problem, I started talking about a problem that I was interested in. He took offence at this.

E. D. And that was the end of it?

M. F. More or less so. Kolmogorov has left a rather different impression on me. He always had something interesting to say on subjects I was interested in, while Linnik liked to talk only about his own research.

E. D. What do you know about Vinberg?

M. F. Not a lot. I know that he successfully defended his doctoral dissertation.

E. D. Was it on the same subject he had been working on?

M. F. Yes, it was on the same subject or almost on the same subject. He made a good progress, and various people tried to help him, but the opportunity to defend presented itself only recently. Recently, there were several defenses of doctoral dissertations which should have taken place earlier. Senya Gindikin defended his dissertation, and Fuchs defended too.

E. D. So there is a certain progress.

M. F. Both are excellent mathematicians, and it is outrageous that they could not get their degrees earlier. Even when they finally defended, they could not do it in Moscow. Gindikin defended in Tbilisi. Fuchs defended somewhere far from Moscow as well.

E. D. How is our "good friend" Pontryagin? Is he still active in his fight with "Zionists"?

[11] http://en.wikipedia.org/wiki/Yuri_Vasilevich_Prokhorov

M. F. I haven't heard much about him for a long time. Now there is a new generation of "anti-Zionists". I am not sure if Vladimirov[12] qualifies as younger generation but he is very active now too.

E. D. Is he also as virulent as he used to be?

M. F. Rumor has it that he tries to be more moderate to have a better chance of becoming the director of the Steklov Institute.

[12] http://en.wikipedia.org/wiki/Vasilii_Sergeevich_Vladimirov