

Interview with Fridrich Israilevich Karpelevich
September 11, 1989. Moscow, Hotel of the Academy of
Sciences of the USSR, Part 1

E. D.: Please tell me about your first steps in mathematics.

F. Karpelevich: First I joined a school mathematical circle¹ in the ninth grade. Then it was run by Sasha Kronrod. I attended two or three times and then for some reason stopped going. It didn't work for me.

E. D.: Do you remember why?

F. K.: There was something about it that didn't appeal to me. Then when I started attending again a year later, in the tenth grade, you were in charge, and this time I liked it (laughs).

E. D.: And there you became top student, in some sense.

F. K.: Well, I don't know, top or not.... But there were some participants of long standing: Volodya Uspensky, Lyonya Rozenknop, and, if I remember correctly, Kolya Beskin.

E. D.: I don't recall him.

F. K.: Fridlender was there too, but for some reason we became acquainted only in my first year at university.

E. D.: As I recall he wasn't admitted into the University.

F. K.: He wasn't admitted, and I met him only when I came to your apartment for something or other.... In the first year our kruzhok into a seminar for first-year students.... As a participant of the kruzhok I solved a fine problem about coloring maps.

E. D.: I no longer remember the problem.

F. K.: I remember it well. I proved that if for each country the number of boundaries with adjoining countries is divisible by three, then the map can be colored.

E. D.: A nontrivial problem.

F. K.: Yes, it's nontrivial. I used the result that instead of coloring the countries one can color the boundaries. And I thought up a regular procedure for coloring, and a certain lemma whose proof requires an induction. Induction within some contour. You had declared that there

were no theorems relating to this problem that could be proved by induction, that the inductive approach always fails. But I found such a theorem.

E. D.: What else do you remember? Chentsov was there, I think....

F. K.: Yes, Kolya was there.

E. D.: Who else?

F. K.: There was someone called Korst, also with first name Kolya, but he entered the physics faculty, not mathematics and mechanics²

E.D.: Do you know anything about him?

F. K.: He developed a love for volleyball, and was to some extent successful at it. But for some reason---I don't know why exactly---he died fairly young. There was also Smigla.

E. D.: Him I don't remember at all.

F. K.: You don't? All his fingers were missing. He also enrolled in the physics faculty. There was also Erich Balash.

E. D.: Yes, Erich Balash was there. But only for part of the year, I think. After the mathematical olympiad perhaps.

F. K.: Erich had been in the mathematical circle for more than a year when I joined it. An old hand. I was in it for only a year. At school he achieved something out of the ordinary---something related to the Fibonacci sequence.

E. D.: Yes, yes, yes. He made an impression even as a ninth-grader. A specific problem was posed, but he dug deeper and developed a whole theory.

F. K.: Yes, yes, yes, he obtained significant results. Later we studied together, but somewhere in third year he faded. I don't know why. I know nothing of what became of him. There was also---I don't quite recall---Alik Bachelis.

E. D.: Yes, he was there.

F. K.: So, Alik Bachelis. We were in a course together. He completed university. Then taught somewhere, and then seemed to be mainly busy giving private lessons. He defended a dissertation of some kind---on differential equations, I think, or computational mathematics---I no longer remember. Then he somehow dropped from view. Unfortunately, I don't know what happened to him, or where he is. Then there was Yura Boldakov.

E. D.: Yes.... And these were all in your circle?

F. K.: Yes, all in our circle.

E. D.: He worked on iterations.

F. K.: He achieved some beautiful things. However, in his third year he met with an accident: he drowned. And under odd circumstances. The water was only knee-deep. But somehow he managed to drown. To return to Fridlender: in first year you had me give a talk on Cantor's theory of the real numbers, and I read some textbook but I don't recall which one.

E. D.: Neither do I, of course.

F. K.: And then you gave me some coaching.... I came to your apartment--- still on Leningrad Avenue---and Fridlender was there; you were helping him prepare for an exam on analysis...

E.D.: He was not admitted to MSU and I tried to arrange his transfer from another institute.

F.K.: I think that he failed this exam but I am not sure.

E.D.: Next year I arranged a similar exam for Misha Agranovich.³ He was examined by Khinchin, Pavel Sergeevich [Aleksandrov], and---I don't recall the third---Kurosh, maybe. They all supported me and he was accepted. In those days such things could happen. \footnote{An influence of faculty members on admission to MSU vanished a few years later.}

F. K.: I still remember very well that when I came in you were scolding Fridlender for preparing himself poorly, for his poor comprehension.

E. D.: Well, unfortunately scolding rarely helps, generally speaking.

F. K.: If I remember correctly, in my first year I wrote my first paper on pseudo-norms of integers. It was like this: to begin with I was supposed to give a talk on a paper of Mahler, and then rework what he'd done, reprove all of that material, and that was my first paper, which I rewrote and brought to you four times, only to have you keep turning it down.

E. D.: Well, that was good for you.

F. K.: Of course, wonderful! What else do I recall.... Various hiking trips. Spring seminars, which took place in the bosom of nature.

E. D.: To be honest, I don't remember any of that.

F. K.: In the last seminar meeting of the year it would be announced that on some Sunday the whole seminar would make an excursion beyond the town somewhere. There were a number of such trips.

E. D.: Once a year, more or less?

F. K.: Well, it was in spring. As a rule it took place somewhere in the countryside, but in various locales. It was not just for a single year, but went on for five years, as I recall---every year.

E. D.: I understand, I understand. But when you were in your second year new people came.

F. K.: In second year....

E. D.: Alik Yushkevich appeared...

F. K.: Alik Yushkevich came and Alik Berezin. But he didn't attend regularly, although he was there all right. And then Rafka Khasminskii.

E. D.: No, later, I think.

F. K.: You mean he's even younger?

E. D.: Even younger.

F. K.: Yulik Dobrushin also appeared then.

E. D.: Yulik participated in the seminar even in his first year.

F. K.: I don't recall that he participated from the first year.

E. D.: In any case he was in the same year as you?

F. K.: Yes, yes, yes, yes, yes, of course he was there from first year! I clearly remember his expounding Stone's generalization of Weierstrass's theorem. And he expounded it very well. And although you rarely praise speakers for their talks, you did praise Yulik. His talk was excellent.

E. D.: Yes, there exist such people. Kirillov was also there from the very beginning.

F. K.: I didn't meet him then because he arrived after I had gone to Novocherkassk. But when I returned, there he was. At that time he wrote a paper on the action of the orthogonal group on vector fields on the sphere.

E. D.: Let's come back to your scientific biography. First there were all those baby papers and then came your serious work on characteristic roots. That was something that I had been unable to do. It was serious

F. K.: Yes. I didn't understand then, but do now, that that work, while of medium quality rather than being outstanding, was entirely appropriate for a candidate's⁴ dissertation.

E.D.: Anyway you rounded off partial results of Dmitriev and Dynkin published a few years before in *Izvestiya*⁵. At that time *Izvestiya* was not translated into English. The American Mathematical Society has just now sent me a translation. The paper published in 1946 was translated in 1988 in the latest collection of translations from Russian. Now I have a pile of reprints in English.

F. K.: Then it would make sense for a translation of my paper be issued to complement yours.

E. D.: Quite right. But perhaps your paper was already translated earlier. It would be interesting to find out.

F. K.: My paper appeared in *Izvestiya* in 1950 or 1951, if I'm not mistaken.

E. D.: *Izvestiya* was probably not translated back then. You know that the present Chairman of the committee on translations is Mark Freidlin. I was Chair for three years before him. I should point out to him that, if your paper is not yet translated, then it is essential that it be translated.

F. K.: As a matter of fact, some years ago---I don't remember if it was at the end of 1984 or the beginning of 1985---they did write to me to let me know that they intended to translate that paper, and to ask if I had any ideas on further developments.

E. D.: So they did translate it?

F. K.: I don't think they did because I never received anything.

E. D.: That doesn't mean a thing. Well, it's not difficult to find out, even here. When I listen to this recording in America, I'll be reminded of our conversation, and find out.⁶ Well there are more interesting subjects for recording than the timing of translating. Do you recall how you celebrated my twenty-fifth birthday?

F. K.: That I have no recollection of. But I do remember how we celebrated defense of your doctoral dissertation.⁷ Someone had the idea of making models from plasticine and knitting needles of the diagrams of simple root systems---I don't remember who it was, but I do remember how Kolya Chentsov and I ran about Moscow looking for flowers to buy for you. No one would have told you about that....

E. D.: Why exactly was that a problem?

F. K.: Well, we couldn't find any flowers. However, at long last we found a sort of booth, which then had to shut up shop because we bought up all the flowers they had. It was marvelous because when the saleswoman asked Kolya how many he wanted, he said "Give me the lot!"

E. D.: Where did you get the money?

F. K.: Somehow or other we had it.... That wasn't the main thing....

E. D.: That was , in 1951. You were already then in your final undergraduate year.

F. K.: Yes, I was in my fourth year. I graduated in 1952.

E. D.: And the match-stick models? Did you take part in that too?

F. K.: Of course! I even recall that after your defense we all went to your apartment on Leningrad Avenue, and you hung the model of ⁸ up near the ceiling on the chandelier. The important thing was that it was fairly robust, having a double two-needle connection. I also recall that it was Kolya Chentsov and I who presented it to you, and you asked in puzzlement "What's this?" And one of us answered "It's F_4 !", to which you responded "Aha!"

E. D.: Some things I seem to remember, others not.

F. K.: What can one do? That's the way of the world. Well, what else is there to talk about?

E. D.: What did your undergraduate thesis ⁹ consist of? Lie algebras of some sort?

F. K.: The subject matter of my undergraduate thesis developed into that of my candidate's dissertation. It concerned semisimple subalgebras. That was already genuine research.

E. D.: Well of course.

F. K.: In my fourth year you had me give a talk on Gantmacher's reworking of Cartan's work on the classification of involutory automorphisms.

E. D.: Which is, of course, just the classification of real forms.

F. K.: Yes, that was the classification of real forms of complex algebras. I did it. I read the paper and translated everything into the language of simple roots, as you doubtless recall. The result was a very elegant piece of work. I talked about it several times at the seminar, and then by way of a topic for my undergraduate thesis you suggested finding all subalgebras. So I looked for them....

E. D.: And you did prove a general theorem.

F. K.: As to that general theorem, the canonical embedding theorem, well, that's what it's called now. Recently Erik [Vinberg] and Onishchik published a book on algebraic groups, and they call it precisely that: the canonical embedding theorem for algebraic groups. Well, of course, that theorem is fundamental. I came to it as follows: At first I thought it rather trivial, and that the chief difficulty would be to find all possible extensions. I struggled for a long time with that, and only when I sat down to write it up, and order my thoughts, did I find it resisting proof, and only by sorting through special cases did I manage to prove it. In my undergraduate thesis one case remained open---I was then unable to finish off the calculation---until finally the summer before I went to Novochoerkassk, I reduced it to a certain integral, which I was able to evaluate.

E. D.: When you finished high school you suddenly declared that you didn't want to enter university, but wanted to find work, and I was very surprised and tried to dissuade you. I don't remember now what your reasoning was back then.

F. K.: Well, you must understand that it was a issue of a simple material nature, to do with my growing up in a very needy family.

E. D.: Tell a little about your family.

F. K.: Well, there were three of us, siblings, and in addition Grandma. Mama was a quality control worker in a factory, and she was paid 70 rubles a month. Of course, that's not today's rubles, but it's not much more.

E. D.: And your father?

F. K.: My father divorced my mother when I was seven and my sister a one-year-old, and we got practically no help from him, except for a little alimony that came irregularly.

¹ Mathematical circles ["kruzhki"] and olympiads for gifted school students worked at Moscow State University ["MSU"]. "Kruzhki" were lead by the University students.

² "Fizfac" and "Mekhmat" are common Russian abbreviations for Faculty of Physics and Faculty of Mechanics and Mathematics, respectively.

³ M. S. Agronovich graduated from MSU [Moscow State University] in 1953. For his biographical data see, Mikhail Semenovich Agronovich (on his 70th birthday, Russian Mathematical Surveys (2001),56(4):777.

⁴ The "candidate" degree is roughly equivalent to the Ph. D.

⁵ Izvestiya Akademii Nauk SSSR, Seriya Matematicheskaya.

⁶ The paper of Karpelevich as well as the paper of Dmitriev and Dynkin were published in Amer.Math.Soc. Transl.(2) Vol. 140, 1988.

⁷ Doctoral degree is a higher degree attesting to substantial research achievements beyond the degree of candidate.

⁸ F_4 is one of the exceptional simple Lie groups described by a Dynkin diagram

⁹ An undergraduate thesis ["Diplomnaya rabota"] is roughly equivalent to a Master's thesis.